of generality that the altitude from A to BC has length 1. The above equations then imply that the line segments perpendicular to BC from K, L, M, N equal $\frac{2}{7}, \frac{1}{2}, \frac{2}{7}, \frac{1}{5}$, respectively. Thus

$$\frac{\text{Area}(KLM)}{\text{Area}(KMN)} = \frac{\frac{1}{2} - \frac{2}{7}}{\frac{2}{7} - \frac{1}{5}} = \frac{5}{2},$$

and therefore

$$\frac{\text{Area}(KLM)}{\text{Area}(KLMN)} = \frac{5}{7}.$$

Also solved by ŠEFKET ARSLANAGIĆ, University of Sarajevo, Sarajevo, Bosnia and Herzegovina; OLIVER GEUPEL, Brühl, NRW, Germany; JOHN G. HEUVER, Grande Prairie, AB; VÁCLAV KONEČNÝ, Big Rapids, MI, USA(2 solutions); and MICHAEL PARMENTER, Memorial University of Newfoundland, St. John's, NL.

3645. [2011 : 235, 238] *Proposed by José Luis Díaz-Barrero and Juan José Egozcue, Universitat Politècnica de Catalunya, Barcelona, Spain.*

Let a, b, and c be positive numbers such that $a^2 + b^2 + c^2 + 2abc = 1$. Prove that

$$\sum_{\text{cyclic}} \sqrt{a\left(\frac{1}{b} - b\right)\left(\frac{1}{c} - c\right)} \ > \ 2\,.$$

I. Solution by Arkady Alt, San Jose, CA, USA.

Observe that $0 < a, b, c < 1$ so that $abc \neq 1$. The inequality is equivalent to

$$a\sqrt{(1-b^2)(1-c^2)} + b\sqrt{(1-c^2)(1-a^2)} + c\sqrt{(1-a^2)(1-b^2)} > 2\sqrt{abc}. \quad (1)$$

Since

$$(1-b^2)(1-c^2) = 1 - b^2 - c^2 + b^2c^2 = a^2 + 2abc + (bc)^2 = (a+bc)^2,$$

and, similarly, $(1-c^2)(1-a^2) = (b+ca)^2$ and $(1-a^2)(1-b^2) = (c+ab)^2$, the left side is equal to

$$\begin{aligned} a(a+bc) + b(b+ca) + c(c+ab) &= a^2 + b^2 + c^2 + 3abc \\ &= 1 + abc > 2\sqrt{abc} \end{aligned}$$

by the Arithmetic-Geometric Means Inequality.

II. Solution using ideas from Šefket Arslanagić, University of Sarajevo, Sarajevo, Bosnia and Herzegovina; Oliver Geupel, Brühl, NRW, Germany; Salem Malikić, student, Simon Fraser University, Burnaby, BC; and Albert Stadler, Herrliberg, Switzerland.